Answer of 12 by Xu Min

- 1.

$$EX = \int_{-\infty}^{+\infty} xf(x)dx = \int_a^b x\frac{x}{b-a}dx = \frac{a+b}{2}$$

$$EX^2 = \int_{-\infty}^{+\infty} x^2 f(x)dx = \int_a^b \frac{x^2}{b-a}dx = \frac{a^2+ab+b^2}{3}$$

$$a+b = 2\overline{x}, (b-a)^2 = 12s^2, \hat{a} = \overline{x} - \sqrt{3}s, \hat{b} = \overline{x} + \sqrt{3}s$$

- 2.

$$\ln L = n\ln\theta + (\theta-1)\ln(\prod x_i^{\theta-1})$$

$$\frac{dlnL}{d\theta} = \frac{n}{\theta} + ln(\prod x_i)$$

$$\hat{\theta} = -\frac{n}{ln(\prod lnX_i)}$$

- 3.

$$\ln L = -\frac{n}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\Sigma(x_i-\mu)^2$$

$$\frac{\partial \ln L}{\partial \mu} = \frac{1}{\sigma^2}\Sigma(x_i-\mu) = 0$$

$$\frac{\partial \ln L}{\partial \sigma^2} = -\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4}\Sigma(x_i-\mu)^2 = 0$$

$$\hat{\mu} = \frac{1}{n}\Sigma x_i = \bar{x}, \hat{\sigma^2} = \frac{1}{n}\Sigma(x_i-\bar{x})^2$$

$$\hat{P}(x-t) = \Phi(\frac{x-\hat{\mu}}{\sigma})$$

- 4.

$$\hat{\mu} = \frac{1}{n}\Sigma x_i = \bar{x} = 997.1$$

$$\hat{\sigma^2} = \frac{1}{n}\Sigma(x_i - \bar{x})^2 = 15574.29$$

$$P(x > 1300) = 1 - P(x < 1300) = 1 - \Phi(2.4271) = 0.008$$

- 6.

$$E(x_{i+1} - xi)^2 = D(x_{i+1} - xi) = 2\sigma^2$$

$$\sigma^2 = cE[\sum_{i=1}^{n-1}(x_{i+1} - xi)^2] = c(n-1)2\sigma^2, c = \frac{1}{2(n-1)}$$

- 7.

$$E(\hat{\theta}^2) = D(\hat{\theta}) + (E(\hat{\theta}))^2 = D(\hat{\theta}) + \theta^2 > \theta^2$$

$$unbiased$$

- 9.

$$\ln L = -\frac{n}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\Sigma(x_i)^2$$

$$\frac{\partial \ln L}{\partial \sigma^2} = -\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4}\Sigma x_i^2 = 0, \hat{\sigma^2} = \frac{1}{n}\Sigma x_i^2$$

$$E(\hat{\sigma^2}) = E(\frac{1}{n}\Sigma x_i^2) = \sigma^2, unbiased$$

$$D(\hat{\sigma^2}) = (\frac{1}{n}\sigma^2)^2 D\chi^2(n) = \frac{2\sigma^4}{n}$$

$$\forall \varepsilon > 0, 1 \geq P(|\hat{\sigma^2} - E(\hat{\sigma^2})| < \varepsilon) \geq 1 - \frac{D\hat{\sigma^2}}{\varepsilon^2} = 1 - \frac{2\sigma^4}{n\varepsilon^2}$$

$$\lim_{n\to+\infty} P(|\hat{\sigma^2} - \sigma^2| < \varepsilon) = 1$$