Answer of 10 by Xu Min

SIX

- 1.

$$EX = \int_{-\infty}^{+\infty} xf(x)dx = \int_{0}^{+\infty} x\frac{x^m}{m!}e^{-x}dx$$

$$= \frac{1}{m!}\int_{0}^{+\infty} x^{m+2-1}e^{-x}dx = m+1$$

$$EX^2 = \int_{-\infty}^{+\infty} x^2 f(x)dx = \int_{0}^{+\infty} x^2\frac{x^m}{m!}e^{-x}dx$$

$$= \frac{1}{m!}\int_{0}^{+\infty} x^{m+3-1}e^{-x}dx = (m+2)(m+1)$$

$$DX = EX^2 - (EX)^2 = (m+2)(m+1) - (m+1)^2 = m+1$$

$$P(0 < X < 2(m+1)) = P(-(m+1) < X - (m+1) < (m+1))$$

$$= P(|X - EX| < (m+1)) \geq 1 - \frac{DX}{(m+1)^2} = 1 - \frac{m+1}{(m+1)^2} = \frac{m}{m+1}$$

- 2.

$$\mu_n \sim B(n, \frac{1}{2}), E(\mu_n) = np = \frac{n}{2}, D(\mu_n) = npq = \frac{n}{4}$$

$$P(0.4 < \frac{\mu_n}{n} < 0.6) = P(0.4n < \mu_n < 0.6n)$$

$$= P(|\mu_n - 0.5n| < 0.1n) = P(|\mu_n - E(\mu_n)| < 0.1n) \geq 0.9$$

$$P(|\mu_n - E(\mu_n)| < 0.1n) \geq 1 - \frac{D(\mu_n)}{(0.1n)^2} = 1 - \frac{0.25n}{0.01n^2}, n \geq 250$$

$$P(0.4 < \frac{\mu_n}{n} < 0.6) = P(-0.1n < \mu_n - 0.5n < 0.1n) = P(|\mu_n - 0.5n| < 0.1n)$$

$$= P(|\frac{\mu_n - E(\mu_n)}{\sqrt{D(\mu_n)}}| < \frac{0.1n}{\sqrt{D(\mu_n)}}) = P(|\frac{\mu_n - E(\mu_n)}{\sqrt{D(\mu_n)}}| < 0.2\sqrt{n})$$

$$\approx \Phi(0.2\sqrt{n}) - \approx \Phi(-0.2\sqrt{n}) = 2\Phi(0.2\sqrt{n}) - 1 \geq 0.9, n \geq 68$$

- 3.

$$X \sim B(n,p), n = 6000, p = \frac{1}{6}$$

$$P(|\frac{x}{n} - \frac{1}{6}| < 0.01) = P(|\frac{X - np}{\sqrt{np(1-p)}}| < \frac{0.01n}{\sqrt{np(1-p)}})$$

$$= P(|\frac{X - np}{\sqrt{np(1-p)}}| < \frac{0.01 * 6000}{\sqrt{6000 * \frac{1}{6} * \frac{5}{6}}})$$

$$\approx \Phi(2.078) - \Phi(-2.078) = 2\Phi(2.078) - 1 = 0.96$$

- 4.

$$T = X = X_1 + X_2 + \dots X_{30}$$

$$\mu = EX_i = \frac{1}{\lambda} = 10, \sigma^2 = DX_i = \frac{1}{\lambda^2} = 100$$

$$P(T > 350) = 1 - P(T \leq 350) = 1 - P(\frac{T - n\mu}{\sqrt{n}\sigma} < \frac{350 - n\mu}{\sqrt{n}\sigma})$$

$$\approx 1 - \Phi(\frac{350 - 300}{\sqrt{30} * 10}) = 1 - \Phi(0.91) = 0.1814$$

- 5.

$$X \sim B(n,p), n = 200, p = 0.05$$

$$X_i = 1, use\ i\ in\ telephone\ i = 1, 2, \ldots, 200$$

$$X = X_1 + X_2 + \ldots X_{200}$$

$$PX_i = 1 = p = 0.05, PX_i = 0 = 1 - p = 0.95$$

$$P(X \leq N) = 0.9$$

$$P(\frac{X - np}{\sqrt{np(1-p)}} \leq \frac{N - np}{\sqrt{np(1-p)}})$$

$$\approx \Phi(\frac{N - np}{\sqrt{np(1-p)}}) = \Phi(\frac{N - 10}{\sqrt{9.5}}) = 0.9$$

$$N = 14$$