Answer of the Eighth HW by Xu Min

THREE

- 17.

$$A = \int_0^1 dx \int_0^{2x} dy = 1$$

$$f(x, y) = 1, 0 \le x \le 1, 0 \le y \le 2x$$

$$f_X(x) = \int_{-\infty}^{\infty} f(x, y)dy = 2x, 0 \le x \le 1$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x, y)dx = 1 - \frac{y}{2}, 0 \le y \le 2$$

$$f_{x|y}(x|y) = \frac{f(x, y)}{f_Y y} = \frac{2}{2-y}, \frac{y}{2} \le x \le 1$$

$$f_{y|x}(y|x) = \frac{f(x, y)}{f_X x} = \frac{1}{2x}, 0 \le x \le 2x$$

- 19.

$$f_X(x) = \int_{-\infty}^{\infty} f(x, y)dy = 2x, 0 \le x \le 1$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x, y)dx = 1 - |y|, -1 \le y \le 1$$

- 20. (1)

$$f(x, y) = \frac{d^2 F(x, y)}{dxdy} = \frac{xe^{-x}}{(1+y)^2}$$

(2)

$$F_X(x) = F(x, +\infty) = 1 - (1+x)e^{-x}, x > 0$$

$$F_Y(y) = F(+\infty, y) = \frac{y}{1+y}, y > 0$$

$$Independent.$$

- 25. (1)

$$f_X(x) = \frac{1}{\sigma_1\sqrt{2\pi}} e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}}$$

$$f(x,y) = f_X(x) f_Y(y) = \frac{1}{\pi} e^{-\frac{x^2+y^2}{2}}, -\infty < x < \infty, y > 0$$

(2)

$$x = r\cos\theta, y = r\sin\theta$$

$$P(A) = \int\int_{y-x\geq 0, y\geq 0} f(x,y)dxdy = \int_0^{\infty}\int_{\frac{\pi}{4}}^{\pi} \frac{1}{\pi} e^{-\frac{r^2}{2}} r dr d\theta = \frac{3}{4}$$

- 29.

$$F_X(x) = F(x, +\infty) = 1 - e^{-0.01x}, x > 0$$

$$F_Y(y) = F(+\infty, y) = 1 - e^{-0.01y}, y > 0$$

$$F(x,y) = F_X(x) F_Y(y)$$

$$Independent.$$

$$P(x > 120, Y > 120) = P(x > 120)P(y > 120) = (1 - F_X(120))(1 - F_Y(120)) = e^{-2.4}$$

FIVE

- 13.

$$E(X+Y) = \int_0^1 \int_0^x (x+y)2dxdy = 1$$

$$E(XY) = \int_0^1 \int_0^x 2xydxdy = \frac{1}{4}$$

- 18.

$$x = r\cos\theta, y = r\sin\theta$$

$$E(XY) = \int_0^1 \int_0^{2\pi} \frac{r^3 \sin 2\theta}{\pi} dr d\theta = 0$$

$$E(X) = \int\int_{x^2+y^2\le 1} x\frac{1}{\pi}dxdy = 0$$
$$E(Y) = \int\int_{x^2+y^2\le 1} y\frac{1}{\pi}dxdy = 0$$
$$\rho_{XY} = \frac{COV(X,Y)}{\sqrt{DX}\sqrt{DY}} = 0$$
$$Uncorrelated.$$

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)dy = \frac{2}{\pi}\sqrt{1-x^2}, |x| \le 1$$
$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)dx = \frac{2}{\pi}\sqrt{1-y^2}, |y| \le 1$$
$$f(x,y) \ne f_X(x)f_Y(y)$$
$$Unindependent.$$

- 22.

$$COV(X,Y) = E(XY) - 0 = E[(X_1+X_2+X_3)(X_2+X_3+X_4)] = E(X_2^2+X_3^2) = 2\sigma^2$$

$$D(X) = D(X_1+X_2+X_3) = DX_1+DX_2+DX_3 = 3\sigma^2$$

$$\rho_{XY} = \frac{COV(X,Y)}{\sqrt{DX}\sqrt{DY}} = \frac{2}{3}$$

- 23. (1)

$$\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} a\sin(x+y)dxdy = a\int_0^{\frac{\pi}{2}}(\sin y+\cos y)dy = 2a = 1, a = \frac{1}{2}$$

(2)

$$EX = \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} xf(x,y)dxdy = a\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} x\sin(x+y)dxdy = \frac{\pi}{4}$$

$$EX^2 = \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} x^2 f(x,y)dxdy = a\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} x^2\sin(x+y)dxdy = \frac{1}{2}(\frac{\pi^2}{4}+\pi-4)$$

$$DX = EX^2 - (EX)^2 = \frac{\pi^2}{16} + \frac{\pi}{2} - 2, EY = \frac{\pi}{4}, DY = \frac{\pi^2}{16} + \frac{\pi}{2} - 2$$

(3)

$$E(XY) = \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} xyf(x,y)dxdy = a\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} xy\sin(x+y)dxdy = \frac{\pi}{2} - 1$$

$$COV(X,Y) = E(XY) - E(X)E(Y) = \frac{\pi}{2} - \frac{\pi^2}{16} - 1$$

$$\rho_{XY} = \frac{COV(X,Y)}{\sqrt{DX}\sqrt{DY}} = \frac{8\pi - 16 - \pi^2}{\pi^2 + 8\pi - 32}.$$