习题七

1.**解**：由于$X_1, X_2, \cdots, X_n$同服从(0–1)分布，$EX_i = p, DX_i = p(1-p)$，又$X_1, X_2, \cdots, X_n$相互独立，则
$E\overline{X} = E(\frac{1}{n}\sum_{i=1}^n X_i) = \frac{1}{n}E(\sum_{i=1}^n X_i) = \frac{1}{n}\sum_{i=1}^n E(X_i) = p,$
$D\overline{X} = D(\frac{1}{n}\sum_{i=1}^n X_i) = \frac{1}{n^2}\sum_{i=1}^n D(X_i) = \frac{p(1-p)}{n},$
$\sum_{i=1}^n X_i = n\overline{X}, \sum_{i=1}^n (X_i - p) = n(\overline{X} - p).$
计算ES^2
$ES^2 = E[\frac{1}{n-1}\sum_{i=1}^n (X_i - \overline{X})^2] = E\{\frac{1}{n-1}\sum_{i=1}^n [(X_i - p) - (\overline{X} - p)]^2\}$
$= \frac{1}{n-1}[\sum_{i=1}^n E(X_i - p)^2 - nE(\overline{X} - p)^2] = \frac{1}{n-1}[\sum_{i=1}^n DX_i - nD\overline{X}] = p(1-p)$

2.**解**：根据题设条件X_i的分布律为$P\{X_i = k_i\} = \frac{e^{-\lambda}\lambda^{k_i}}{k_i!}(k_i = 0, 1, 2, \cdots).$
又$X_1, X_2, \cdots, X_n$相互独立，于是$(X_1, X_2, \cdots, X_n)$的分布律为
$P\{X_1 = k_1, X_2 = k_2, \cdots, X_n = k_n\} = \Pi_{i=1}^n P\{X_i = k_i\} =$
$\frac{e^{-n\lambda}\lambda^{\sum_{i=1}^n k_i}}{k_1!k_2!\cdots k_n!} \quad (k_i = 0, 1, 2, 3, \cdots; i = 1, 2, \cdots, n).$

3.**解**：根据正态总体的样本的线性函数的性质
$\overline{X} = \frac{1}{n}\sum_{i=1}^n X_i \sim N(\mu, \frac{\sigma^2}{n}),$
于是$\overline{X} \sim N(20, \frac{1.5^2}{25}) = N(20, 0.3^2),$
所以$P\{19.6 < \overline{X} < 20.3\} = P\{-\frac{4}{3} < \frac{\overline{X} - 20}{0.3} < 1\}$
$= \Phi(1) - \Phi(-1.33) = 0.8413 - 0.0918 = 0.7495$

4.**解**：由题意得$E(X) = \mu = 0, D(X) = \sigma^2 = 0.3^2$
又$X_1, X_2, \cdots, X_{10}$相互独立,于是
$E(\sum_{i=1}^{10} X_i^2) = \sum_{i=1}^{10} E(X_i^2) = \sum_{i=1}^{10} E[X_i - E(X)]^2 = \sum_{i=1}^{10} D(X_i) = 10 \times 0.3^2 = 0.9$
$\frac{1}{\sigma^2}\sum_{i=1}^{10}(X_i - \mu)^2 = \frac{1}{0.3^2}\sum_{i=1}^{10} X_i^2 \sim \chi^2(10)$
$D(\sum_{i=1}^{10} X_i) = 0.162$
$P\{\sum_{i=1}^{10} X_i > 1.44\} = P\{\frac{1}{0.3^2}\sum_{i=1}^{10} X_i > \frac{1.44}{0.3^2}\}$
$= P\{\chi^2(10) > 16\} = 1 - P\{\chi^2(10) \leq 16\}$
查表得$\chi^2_{0.90}(10) = 15.987$
所以$P\{\sum_{i=1}^{10} X_i > 1.44\} = 1 - 0.90 = 0.1$
即$P\{\sum_{i=1}^{10} X_i > 1.44\} = 1 - 0.90 = 0.1, E(\sum_{i=1}^{10} X_i^2) = 0.9, D(\sum_{i=1}^{10} X_i) = 0.162$

5.**解**：查表得$\chi^2_{0.90}(10) = 15.987, \chi^2_{0.95}(50) = 67.2206$

$t_{0.95}(7)=1.8946, t_{0.05}(6)=-t_{0.95}(6)=-1.9432$
$F_{0.99}(10,9)=5.26, F_{0.10}(4,6)=\frac{1}{F_{0.90}(4,6)}=0.2494$

6.**解:** 因为$\alpha(\overline{X}-\mu_1)+\beta(\overline{Y}-\mu_2)\sim N(0,(\frac{\alpha^2}{m}+\frac{\beta^2}{n})\sigma^2)$,
所以$\frac{\alpha(\overline{X}-\mu_1)+\beta(\overline{Y}-\mu_2)}{\sqrt{(\frac{\alpha^2}{m}+\frac{\beta^2}{n})\sigma^2}}\sim N(0,1)$,
又$\frac{(m-1)S_1^2}{\sigma^2}\sim\chi^2(m-1), \frac{(n-1)S_2^2}{\sigma^2}\sim\chi^2(n-1)$，且相互独立，从而由$\chi^2$分布的性质得
$\frac{(m-1)S_1^2}{\sigma^2}+\frac{(n-1)S_2^2}{\sigma^2}\sim\chi^2(m+n-2)$,
再由$\frac{\alpha(\overline{X}-\mu_1)+\beta(\overline{Y}-\mu_2)}{\sqrt{(\frac{\alpha^2}{m}+\frac{\beta^2}{n})\sigma^2}}$与$\frac{(m-1)S_1^2}{\sigma^2}+\frac{(n-1)S_2^2}{\sigma^2}$相互独立，故由t分布的定义知
$Z=\frac{\alpha(\overline{X}-\mu_1)+\beta(\overline{Y}-\mu_2)}{\sqrt{\frac{(m-1)S_1^2+(n-1)S_2^2}{m+n-2}}\sqrt{\frac{\alpha^2}{m}+\frac{\beta^2}{n}}}\sim t(m+n-2)$.