第三次习题解答 by 梁艳

- 31.解： 设A =产品获准出厂，$\bar{A}$ =产品未获准出厂，B =产品是合格品，$\bar{B}$ =产品是不合格品，根据题设条件知$P(B)=0.96$， $P(\bar{B})=0.04$, $P(A\mid B)=0.98$，$P(A\mid\bar{B})=0.05$，利用贝叶斯公式得所求概率为

$$\begin{aligned} P(A|B) &= \frac{P(B)P(A|B)}{P(B)P(A|B)+P(\bar{B})P(A|\bar{B})} \\ &= \frac{0.96\times 0.98}{0.96\times 0.98+0.04\times 0.05}=0.9979 \\ P(\bar{B}|\bar{A}) &= \frac{P(\bar{B})P(\bar{A}|\bar{B})}{P(\bar{B})P(\bar{A}|\bar{B})+P(B)P(\bar{A}|B)} \\ &= \frac{0.04\times 0.95}{0.04\times 0.95+0.96\times 0.02}=0.6643 \end{aligned}$$

- 35. 解：设A_i表示开关闭合$(i=1,2,3,4,5,6)$ ，A表示L_2K_2为通路，则

$$\begin{aligned} P(A) &= 1-(P(\bar{A}_1)+P(A_1\bar{A}_2\bar{A}_6)+P(A_1\bar{A}_6A_2\bar{A}_3\bar{A}_4\bar{A}_5)) \\ &= 1-\{1-p+p[(1-p)((1-p)+p(1-p)(1-p^2))]\} \\ &= p^2+p^3-2p^5+p^6 \end{aligned}$$

- 38.证：⟨法一⟩

$$P(B|A)=\frac{P(AB)}{P(A)}, P(B|\bar{A})=\frac{P(\bar{A}B)}{P(\bar{A})}=\frac{P(B)-P(AB)}{1-P(A)}$$

$$\because P(B|A)=P(B|\bar{A}), \therefore P(AB)-P(A)P(AB)=P(A)P(B)-P(A)P(AB)$$

$$\therefore P(AB)=P(A)P(B)$$

⟨ 法二⟩

$$\begin{aligned} P(B)=P(AB)+P(\bar{A}B) \\ =P(B|A)P(A)+P(B|\bar{A})P(\bar{A}) \\ =P(B|A)(P(A)+P(\bar{A})) \end{aligned}$$

- 39. 设需要n门高射炮同时射击才能以99% 的把握击中来犯的一架敌机，令A_i =第i门炮击中敌机，A =敌机被击中，则

$$A = A_1 + A_2 + \cdots + A_n = \sum A_i$$

$$P(A) = P(\sum A_i) = 1 - P(\sum \bar{A}_i)$$

$$1 - P(\bar{A}_1 \bar{A}_2 \cdots \bar{A}_n) = 1 - P(\bar{A}_1)P(\bar{A}_2) \cdots P(\bar{A}_n)$$

  即$1-(0.5)^n \geq 0.99$ 于是得 $0.01 \geq 0.5^n, \log 0.01 \geq \log 0.5n, n \geq \frac{lg0.01}{lg0.5} \approx 6.644$ 取n=7。得$n \geq 7$,至少需要7门炮

- 40.解：设A =飞机被击落，B_i =飞机被i个人击中，A_i =第i个人射击击中飞机$(i = 1, 2, 3)$

  由题设条件知，$P(A_1) = 0.4, P(A_2) = 0.5, P(A_3) = 0.7$，$A_1$,$A_2$,$A_3$相互独立，则

  $P(A|B_1) = 0.2, P(A|B_2) = 0.6, P(A|B_3) = 1,$

  $B_1 = A_1\bar{A}_2\bar{A}_3 + \bar{A}_1 A_2 \bar{A}_3 + \bar{A}_1\bar{A}_2 A_3,$

  $B_2 = A_1 A_2 \bar{A}_3 + A_1 \bar{A}_2 A_3 + \bar{A}_1 A_2 A_3, B_3 = A_1 A_2 A_3$

  由概率的可加性和事件的独立性得

  $P(B_1) = P(A_1\bar{A}_2\bar{A}_3) + P(\bar{A}_1 A_2\bar{A}_3) + P(\bar{A}_1\bar{A}_2 A_3)$

  $= P(A_1 P(\bar{A}_2)P(\bar{A}_3) + P(\bar{A}_1)P(A_2)P(\bar{A}_3) + P(\bar{A}_1)P(\bar{A}_2)P(A_3) = 0.36$

  $P(B_2) = P(A_1 A_2 \bar{A}_3) + P(A_1\bar{A}_2 A_3) + P(\bar{A}_1\bar{A}_2 A_3) =$

  $P(A_1)P(A_2)P(\bar{A}_3) + P(A_1)P(\bar{A}_2)P(A_3) + P(\bar{A}_1)P(\bar{A}_2)P(A_3) = 0.41$

  $P(B_3) = P(A_1 A_2 A_3) = P(A_1)P(A_2)P(A_3) = 0.14$

  由全概率公式得

$$P(A) = \sum P(B_i)P(A|B_i) = 0.36 \times 0.2 + 0.41 \times 0.6 + 0.14 \times 1 = 0.458$$